Distributed Optimization and Flight Control Using Collectives

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What is a collective?

- **Collective**: A group of self-interested, learning agents that act together to maximize a global objective.

Examples of collectives are all around us,
- Economies, corporations, examples from nature
- Emergent behavior
What is a collective?

- New technologies are enabling a wide variety of artificial collectives
  - Advances in biology, robotics, software, aerospace systems
  - Design local rewards to obtain desired global performance

- A collective must include:
  - a *world utility* to measure the overall system performance
  - a *private utility* that each agent tries to maximize
Collectives for aerospace systems

- Aerospace systems continue to grow in complexity while demanding optimal performance
  - Systems are comprised of many interacting components
  - System objective is usually well defined

- Collectives have potential application to both the design and control of aerospace systems

Courtesy of the Boeing Company

Courtesy of the Harvard-Smithsonian Center for Astrophysics
Collectives for flight vehicle control - Motivation

High-altitude long endurance vehicles experience significant aeroelastic challenges
Collectives for flight vehicle control - Approach

Utilize large numbers of small, simple, trailing-edge devices to control rigid body and flexible mode vehicle dynamics
Selected Relevant Literature

• Distributed optimization for design
  – System level coordinates distributed subspace optimizations
  – Several architectures exist
    • CO (Braun, et al.), BLISS (Sobieski)
  – Approaches typically exploit smoothness of the subspace optimum

• Distributed control
  – Very wide and active field
    • Almost all applied to control of multiple vehicles
    • Decentralized optimization (Inalhan, et al.)
    • Multi-agent learning (Wellman, Littman)
    • Swarm intelligence (Bonabeau)
  – Approaches operate on the actions of the various agents rather than their strategies
Selected Relevant Literature

• Flight control with distributed effectors
  - Large body of research on different types of effectors
  - Many applications identified including aircraft, rotorcraft, and wind turbines
  - Little research on control concepts and architectures
    • Hierarchical approach (Lyshevski)
    • Linearized control (Raney)
      • Approaches are not general
  - Little research on closed loop experiments
Scope of presentation

• Introduce collectives

• Application of collectives to optimization

• Formulate controller synthesis as an optimization

• Present two experimental test beds

• Summary and future directions
Illustrative Example

- Consider a simple two-player game
  - Players attempt to maximize their reward

- Players learn to cooperate through repeated plays
  - Start with initial probability (strategy)
  - Select moves by sampling probabilities
  - Update probabilities using received rewards

- Probability collectives provides a mathematical framework for this process
• Given a probability distribution, \( P(x) \)
  - Want to quantify how “uncertain” you are that you will observe a value \( x \) generated from \( P(x) \)

• Information theory quantifies this as,

\[
\text{uncertainty}(x) = -\ln P(x)
\]
• The expected uncertainty of a distribution is the Shannon entropy

\[ S(P) = -\sum_{x \in X} P(x) \ln P(x) \]

• This is concave with infinite gradient and zero value at the edges
  – Maximum for uniform distribution, zero for peaked
Now we can formalize our illustrative example

Given a current level of uncertainty $s$

Each player searches for the probability distribution $q$
that minimizes *expected utility* $E_q[G(x)]$, and is consistent with $S(q) = s$
Probability Collectives Theory

- Formally this is written as

\[
\begin{align*}
\text{minimize} & \quad E_q[G(x)] \\
\text{subject to} & \quad S(q) = s
\end{align*}
\]

- So we must find the critical point of the Lagrangian

\[
L(q,T) = E_q[G(x)] + T[s - S(q)]
\]

i.e., find the q and T such that \( \partial L/\partial q = \partial L/\partial T = 0 \)

- Deep connections with statistical physics (L is “free energy” in mean-field theory), economics, gradient-based optimization
Probability Collectives Theory - Procedure

• Procedure
  1. Specify a value of T, the “temperature”
  2. Minimize the Lagrangian with respect to q at specified T
     - Sample the system
     - Update the probabilities
  3. Reduce the value of T and repeat

• Agent sampling and updating are performed independently, but probability distributions are coupled through the expectation

• Role of temperature
  - Controls trade between exploration versus exploitation
Probability Collectives Theory

- Independent agents are assigned to the variables
  - Joint distribution $q$ in the Lagrangian is a product,
    \[ q = \prod_i q_i(x_i) \]

- The resulting optimization for each agent becomes,
  \[
  \min L_i = \sum_{q_i} E\left[ G(x_i, x_{(i)}) \mid x_i = j \right] q_i(j) + T \sum_{j \in X_i} q_i(j) \ln q_i(j)
  \]

- Many nice features (convex, single minimum in the interior) including an analytical gradient (and Hessian)
  - Variety of distributed solution approaches can be developed
  - Most effective approach uses second order descent
• Expectations estimated using Monte-Carlo sampling or computed analytically
  – Estimation introduces error addressed by private utilities

• Proper choice of private utilities reduces bias and variance
  – Bias is the lack of “alignment” between the utilities
  – Variance is the signal-to-noise ratio

• Concept of private utilities differs from traditional approaches to optimization
Example Private Utilities

- The simplest private utility is the world utility
  - Unbiased but has high variance
    \[ g_{TG}(x_i, x_{(i)}) = G(x_i, x_{(i)}) \]

- Wonderful Life Utility (WLU)
  \[ g_{WLU}(x_i, x_{(i)}) = G(x_i, x_{(i)}) - G(CL_{x_i}, x_{(i)}) \]
  where the clamping operator, \( CL_{x_i} \), fixes \( x_i \)
  - Also unbiased but much lower variance

- Theory formalizes these and other utilities and provides for application specific utilities
  - Utilities to exploit agent dependencies
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Consider function $G$ in the variables $x, y$

• **DO NOT:**
  - Search for $x, y$ that optimize the function $G$

• **INSTEAD:**
  - Consider distributions over $x$ and $y$, that optimize the expected value
• Results in a distributed optimization problem with a favorable form

• Suited for noisy, non-linear, multi-modal functions

• Variable dependencies or hierarchies can be exploited

• Connections to other fields
Relationship to other optimization techniques

- Many non-gradient based optimizers use probabilistic operators, but in variable space
  - Particle Swarm
  - Genetic Algorithms

- Gradient based optimization exploits smoothness in the objective function across variable space
  - Probability collectives similarly exploits smoothness in probability space

- Entropy is a barrier function
  - Interior point methods use barrier functions, but in variable space
Solution Algorithm

1. Initialize
2. Sample
3. Evaluate utilities
4. Evaluate objective
5. Compute expected utility
6. Regression
7. Update probabilities
8. Convergence? (Yes/No)
9. Finished
Peaks function

- Mixture of Gaussians

![Graph showing samples and regression](image)

**Iteration 1**
Peaks function

- Mixture of Gaussians

Iteration 5

- Samples & Regression
- Probabilities
• Mixture of Gaussians

Iteration 10

Peaks function

Probabilities $q_x(x)$, $q_y(y)$

Samples & Regression $g_x(x)$, $g_y(y)$

$G(x,y)$
Peaks function

- Mixture of Gaussians

Iteration 15
Peaks function

- Mixture of Gaussians

Iteration 20

- 3D plot showing the function $G(x,y)$
- Graphs showing samples and regression for $g_x(x)$, $g_y(y)$, $q_x(x)$, and $q_y(y)$
Larger-Scale Problems

- Example presented illustrates the approach
  - Larger-scale problems also examined
  - Problems highlight use of private utilities
  - Approach extended to address constrained problems

- Discrete variables with discrete constraints
  - Fleet Assignment
    - 129 variables (with move spaces of 12 or 30), 184 constraints
  - Combinatorial optimization benchmarks
    - Bin packing problems 60 and 120 variables

- Constraint satisfaction problem benchmarks

- Remainder of talk focuses on application to control
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Collectives for Control

- Linear quadratic (LQ) optimal control searches for matrix $K$:

$$\dot{x}(t) = [A]x(t) + [B]u(t)$$

$$\min_{K} \ V = \int_{0}^{\infty} y^{T}[P]y + u^{T}[Q]u \ dt \quad \text{where} \quad y(t) = [C]x(t) + [D]u(t)$$

$$u(t) = -[K]x(t)$$

- Control design searches for optimal policies
  - Policies map measured states to control inputs (linear for LQ)
  - Optimal policies minimize an objective function (quadratic in states and control for LQ)

- Optimization techniques are required for general systems
  - Arbitrary objectives, non-linear dynamics, discrete inputs
  - Robust to noise/disturbance
  - Suited for distributed systems

- Probability collectives provides an approach for accomplishing these goals
Collectives for Control

- Define a policy
  - For example for discrete actions

- Define a value function for single time evolution of the system
  - For example for linear, discrete time

- Control design is an optimization to minimize $V$ with respect to $\theta$

\[
\begin{align*}
u_i &= \begin{cases}
1 & \theta^T Y \geq 1 \\
0 & \text{otherwise}
\end{cases} \\
\theta &\in \{\theta_1, \theta_2, \ldots, \theta_m\}
\end{align*}
\]

Policy definition

\[
V(\theta) = \sum_{k=1}^{T} y(k)^T [P] y(k) + u(k)^T [Q] u(k)
\]

\[
x^{(k+1)} = [A] x^{(k)} + [B] u^{(k)} + [G] w^{(k)}
\]

\[
y^{(k)} = [C] x^{(k)} + [H] v^{(k)}
\]

Value function definition

\[
\text{minimize} \quad V(\theta)
\]

w.r.t \quad \hat{\theta}_i

Resulting optimization
Collectives for Control

• Robust
  – Minimize the objective over all possible instances of noise and disturbance
  – Agents are present that do not update their probability distributions

• Adaptive
  – Address changes in the system over time, either the dynamics or the noise/disturbance
  – No assumption made about the system (e.g. linear, time invariant), probability collectives approach naturally applies

• Distributed
  – Agents make independent choices of actions
  – Strategies (probability distributions) are coupled
Collectives for Control

• Approach described is general since no assumptions made concerning the system

• Alternate formulations also investigated
  – Collocation formulation where optimization is for both the control policies and the states of the system
  – Resulting problem is sparse, which can be exploited by collectives approach using private utilities
  – Applied to example trajectory optimization problem
Scope of presentation

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• Present two experimental testbeds
  – Equipped with multiple simple trailing edge devices
  – Explore distributed control concepts within a single vehicle

• Summary and future directions
Experimental Platforms

• Goal
  – Flexible and rigid body flight vehicle control using multiple small, simple trailing edge devices

• Two experimental platforms developed
  – Flutter suppression experiment
  – Remotely piloted flight vehicle

• Both utilize Miniature-Trailing Edge Effectors
Miniature-Trailing Edge Effectors

Definition
- Small, 1-5% chord, moveable trailing edge devices deflected vertically into the flow

Inspiration
- Gurney flaps developed for race cars in 1970s

Basic Flow Structure
- Stable separation region ahead of flap and pair of counter-rotating vortices aft
- Significant change in section forces with small (or negligible) hinge moment and small drag penalty
Steady aerodynamics

- Significant changes in lift achievable with small devices

Unsteady aerodynamics

- Linear theory appropriate for control
- Other aspects still under investigation
MiTE Actuator Development

Electromechanical vertical sliding flap concept

- Simple
- Low power and voltage
- Lightweight
- Bandwidth up to 40 Hz
- Fritz Prinz and Byong-ho Park

Device used for flutter suppression experiment

Newer designs developed for flight vehicle
Flutter Suppression Experimental Model

Laminated fiberglass plate as primary structure
• Designed for specific flutter speed and frequency

Actuators, sensors and control logic added
• Four MiTEs
• Three sensors
• Supporting electronics based upon embedded technology
System Identification

- System identification performed over range of flow speeds
- Validated analytical model used for control design
- Resulting analytical model parameterized by flow speed

Comparison at 14 m/s

Analytical model block diagram
Root Locus Versus Flow Speed

Flutter at 15.5 m/s and 4 Hz

Aerodynamic Lag Poles

1st Torsion

2nd Bending

1st Bending

5 m/s

20 m/s

5 m/s

Flutter at 15.5 m/s and 4 Hz

Aerodynamic Lag Poles

1st Torsion

2nd Bending

1st Bending

5 m/s

20 m/s
Flutter Suppression Controller Design

- Policy maps sensors to discrete control input
  - Rate gyro and tip acceleration used
  - Single agent with three actions

- Objective is to minimize the measured values

- Objective is evaluated with a single simulation of the analytical model

\[
\begin{align*}
  u &= \begin{cases} 
    +1 & \theta^T Y > +1 \\
    -1 & \theta^T Y < -1 \\
    0 & \text{otherwise}
  \end{cases} \\
  \theta &= \{\theta_1, \theta_2, \ldots, \theta_m\} \\
  Y &= \begin{bmatrix} y_{\text{gyro}} & y_{\text{accel}} \end{bmatrix}
\end{align*}
\]

\[
V(\tilde{\theta}) = \sum_{k=1}^{T} y(k)^T y(k)
\]

Analytical model block diagram
Closed Loop Test Results

Closed Loop Tests Video
Closed Loop Test Results: Summary

- Optimal two parameter controller increased flutter speed by 25%
- Limited by higher frequency dynamics, model fidelity, and increasing amplitude of flow disturbances
Controller Design - Summary

- Controller synthesis performed at various flow speeds
  - Optimal controllers tested to loss of control

- Many different controller designs explored
  - Controllers using additional features stabilized system to even higher speeds

- Collectives compared with two other approaches
  - Linear Quadratic Gaussian for a linearized variant of the system
  - PEGASUS (Ng and Jordan), a direct reinforcement learning approach
Flight Experimental Platform

Remotely piloted flying wing
- 6 foot span, 30 degrees sweep
- Controlled conventionally or with up to 16 MiTEs

Objectives
- Explore distributed control concepts
- MiTEs for primary flight control
- Distributed stability augmentation
- Gust load alleviation
• **Flight Control Architecture**
  – Pilot commands received centrally and broadcast to distributed agents along bus
  – Agents sample local sensors, fuse with pilot commands, and command actuators
  – Agent decision making balances local goals with pilot commands

• **Advantages**
  – Robust
  – Modular
  – Scalable
Distributed Flight Control “Agents”

- Architecture distributes control through agents
  - Composed of actuators, sensors, and command logic
  - Connected to bus providing power and communication
Flight Experimental Platform : Videos

Pre-Flight Overview Video

In-Flight (External) Video

In-Flight (On-board) Video
MegaMiTE Test Objectives

• MiTEs for primary flight control
  – Compare responses due to conventional control surface and MiTEs

• Distributed stability augmentation
  – Outboard agents use feedback of locally sensed vertical accelerations to command MiTEs
  – No explicit coordination between agents
• Captive tests performed to measure response to “aileron” and “elevator” doublets
  – MegaMiTE mounted to car and free to rotate about all three axes
  – Tests performed with conventional surfaces and MiTEs
  – Flight configuration in terms of power, data acquisition, and pilot interface
Primary Flight Control

Longitudinal doublets

- Control Input
- Time, seconds
- Conventional, degrees
- MiTE position × 10

Lateral doublets

- Control Input
- Time, seconds
- Conventional, degrees
- MiTE position × 10

Pitch rate, deg/sec

- Time, seconds

Roll rate, deg/sec

- Time, seconds

Yaw rate, deg/sec

- Time, seconds
Distributed Stability Augmentation

- Outboard agents measure local acceleration
  - Agents integrate and filter acceleration to obtain vertical velocity
  - Apply single gain and fuse with pilot commands

- Captive test configuration
  - Disturbances primarily due to road

![Graph showing running RMS deg/for Yaw Rate vs. Frequency, Hz]

- VIDEO
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Summary

- Collectives for distributed optimization
  - Optimize over probability distributions rather than variables

- Collectives for distributed control
  - Applicable to general systems

- Flight control with distributed effectors
  - Small, simple devices capable of flexible and rigid-body control of flight vehicles
Key Contributions

• **Collectives for distributed optimization**
  – Derived improved Lagrangian minimization techniques
  – Extended collectives approach to address constrained and continuous problems
  – Developed algorithms and software which implement approach on range of optimization problems

• **Collectives for distributed control**
  – Exploited collectives use of probability distributions to address distributed, robust, non-linear control problems
  – Considered multiple control design approaches

• **Flight control with distributed effectors**
  – Developed concept and hardware for distributed flight control
  – Designed, constructed, and tested two experiments to demonstrate potential of approach
Future Directions

• Collectives for optimization and design
  – Improvements to collectives based optimization approach
  – Distributed design
  – Application to systems-of-systems problems where vehicle design is combined with its utilization
  – Robust design
Future Directions

• Collectives for distributed sensing and control
  – Coordinate multiple UAVs
  – Distributed sensing networks
  – Other distributed actuation concepts

• MegaMiTE experimental testbed
  – Potential for robustness to actuator failures
  – Explore other control objectives and develop multi-purpose controllers and

• Further applications for MiTEs
  – Energy extraction by UAVs (Patel)
  – Using MiTEs to dissipate trailing wake vortices (Eaton)
  – Actuators and supporting electronics can easily be incorporated into other vehicles